GCE Examinations

# Further Pure Mathematics Module FP2

Advanced Subsidiary / Advanced Level

Paper A

Time: 1 hour 30 minutes

*Instructions and Information*

Candidates may use any calculator except those with a facility for symbolic algebra and / or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 8 questions.

*Advice to Candidates*

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.

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**1.** A curve has the equation

$$y = x + 2x^2 + 5x^3.$$

Show that the radius of curvature of the curve at the origin is $\frac{1}{\sqrt{2}}$. **(5 marks)**

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**2.** Show that

$$\int_0^{\ln 2} x \operatorname{sech}^2 x \ \mathrm{d}x = \frac{3}{5}\ln 2 - \ln\left(\frac{5}{4}\right). \quad \textbf{(8 marks)}$$

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**3.** *(a)* Prove that

$$\frac{\mathrm{d}}{\mathrm{d}x}(\arcsin 2x) = \frac{2}{\sqrt{1-4x^2}}. \quad \textbf{(3 marks)}$$

Given that

$$\mathrm{f}(x) = 2x \arcsin 2x + \sqrt{1-4x^2},$$

*(b)* show that

$$\mathrm{f}''(x)\left[\mathrm{f}(x) - x\mathrm{f}'(x)\right] = 4. \quad \textbf{(6 marks)}$$

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**4.**

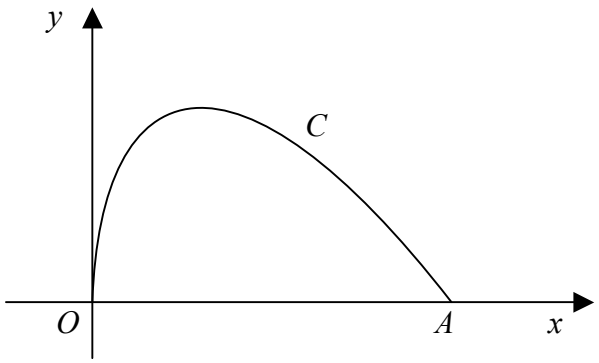



Fig. 1

The parametric coordinates of the curve $C$ shown in Figure 1 are

$$x = t^2, \qquad y = t - \tfrac{1}{3}t^3, \qquad 0 \leq t \leq a.$$

The curve $C$ meets the $x$-axis at the point $A$ where $t = a$.

*(a)* Find the value of $a$. **(2 marks)**

The curve $C$ is rotated through $2\pi$ about $Ox$.

*(b)* Find the surface area of the solid generated. **(8 marks)**

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**5.** *(a)* Using the definitions of $\cosh x$ and $\sinh x$ in terms of $e^x$ and $e^{-x}$, prove that

$$\cosh 2x = 2\cosh^2 x - 1.$$ **(3 marks)**

*(b)* Solve the equation

$$2\cosh 2x = 13\cosh x - 12,$$

giving your answers in terms of natural logarithms. **(7 marks)**

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***Turn over***

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**6.** $x^2 - 10x + 41 \equiv (x+a)^2 + b$.

*(a)* Find the values of the constants $a$ and $b$. **(2 marks)**

*(b)* Show that

$$\int_5^9 \frac{x}{\sqrt{x^2 - 10x + 41}}\ \mathrm{d}x = p\left(\sqrt{2} - 1\right) + q \ln r,$$

stating your values of $p$, $q$ and $r$. **(8 marks)**

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**7.** $$I_n = \int_0^{\frac{\pi}{2}} x^n \cos x\ \mathrm{d}x, \quad n \geq 0.$$

*(a)* Prove that

$$I_n = \left(\frac{\pi}{2}\right)^n - n(n-1)I_{n-2} \quad n \geq 2.$$ **(5 marks)**

*(b)* Hence find the value of $I_4$, giving your answer in terms of $\pi$. **(6 marks)**

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**8.** The rectangular hyperbola $C$ has equation $xy = c^2$, where $c$ is a positive constant.

*(a)* Show that an equation of the tangent to $C$ at the point $P\left(cp, \frac{c}{p}\right)$ is

$$x + yp^2 = 2cp.$$ **(4 marks)**

The tangent to $C$ at $P$ meets the $x$-axis at the point $X$.

The point $Q$ on $C$ has coordinates $\left(cq, \frac{c}{q}\right)$, $q \neq p$ such that $QX$ is parallel to the $y$-axis.

*(b)* Show that $q = 2p$. **(3 marks)**

$M$ is the mid-point of $PQ$.

*(c)* Find, in Cartesian form, an equation of the locus of $M$ as $p$ varies. **(5 marks)**

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**END**

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